Continuity and Differentiability

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q1.

Consider the function $f(x) = [\sin x], x \in [0, \pi]$.

Assertion (A): $f(x)$ is not continuous at $x = \frac{\pi}{2}$.

Reason (R): $\lim_{x \to \pi/2} f(x)$ does not exist.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q2.

Assertion (A): $f(x) = x\left(\frac{1+e^{1/x}}{1-e^{1/x}}\right) (x \neq 0), f(0) = 0$

is continuous at $x = 0$.

Reason (R): A function is said to be continuous at a if both limits are exists and equal to $f(a)$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q3. Assertion (A): $f(x) = |\log x|$ is differentiable at x = 1.

Reason (R): Both log x and -log x are differentiable at x = 1.

Answer : (d) Assertion (A) is false but Reason (R) is true

Q4.

Consider the function $f(x)=\begin{cases} x^2, & x \geq 1 \\ x+1, & x<1 \end{cases}$.

Assertion (A): f is not derivable at $x=1$ as

$$\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x).$$

Reason (R): If a function f is derivable at a point 'a', then it is continuous at 'a'.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q5.

Assertion (A): If $e^{xy}+\log(xy)+\cos(xy)+5=0$, then $\frac{dy}{dx}=-\frac{y}{x}$.

Reason (R): $\frac{d}{dx}(xy)=0 \Rightarrow \frac{dy}{dx}=\frac{-y}{x}$

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q6.

Assertion (A): $\frac{d}{dx}\{\tan^{-1}(\sec x+\tan x)\}$

$$=\frac{d}{dx}\{\cot^{-1}(\operatorname{cosec} x+\cot x)\}, x \in \left(0, \frac{\pi}{4}\right)$$

Reason (R): $\sec^2 x-\tan^2 x=\operatorname{cosec}^2 x-\cot^2 x$

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q7.

Assertion (A): For $x<0, \frac{d}{dx}(\ln|x|)=-\frac{1}{x}$

Reason (R): For $x<0, |x|=-x$

Answer : (d) Assertion (A) is false but Reason (R) is true

Q8.

Assertion (A): If $y = \log_{10} x + \log_e x$, then $\frac{dy}{dx} = \frac{\log_{10} e}{x} + \frac{1}{x}$.

Reason (R): $\frac{d}{dx}(\log_{10} x) = \frac{\log x}{\log 10}$

and $\frac{d}{dx}(\log_e x) = \frac{\log x}{\log e}$.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q9.

Assertion (A): If $y = \frac{1}{4}u^4$ and $u = \frac{2}{3}x^3 + 5$, then $\frac{dy}{dx} = \frac{2}{27}x^2(2x^3 + 15)^3$.

Reason (R): If y is a function of v and v is a function of x, then $\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx}$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q10 **Assertion (A)** The function $f(x) = \sqrt[3]{x}$ is continuous at all x except at $x = 0$.

Reason (R) The function $f(x) = [x]$ is continuous at $x = 2.99$, where [ ] is the

Q11

Q12

$$f(x)=\begin{cases}\sin \pi x, & x<1\\ 0, & x=1\\ \frac{-\sin(x-1)}{x}, & x>1\end{cases}$$

Assertion (A) $f(x)$ is discontinuous at $x=1$.

Reason (R) $f(1)=0$.

Assertion (A) The function $f(x)=\begin{cases}2x, \text{ if } x<0\\ 0, \text{ if } 0\le x\le 1\\ 4x, \text{ if } x>1\end{cases}$ is continuous everywhere except at $x=1$.

Reason (R) Polynomial and constant functions are always continuous.

$$f(x)=\begin{cases}x+\pi, & \text{for } x\in[-\pi,0)\\ \pi\cos x, & \text{for } x\in\left[0,\frac{\pi}{2}\right]\\ \left(x-\frac{\pi}{2}\right)^2, & \text{for } x\in\left(\frac{\pi}{2},\pi\right]\end{cases}$$

Consider the following statements

Assertion (A) The function $f(x)$ is continuous at $x=0$.

Reason (R) The function $f(x)$ is continuous at $x=\pi/2$.

Assertion (A) The function $f(x)=|\cos x|$ is continuous function.

Reason (R) The function $f(x)=\cos|x|$ is a continuous function.

Assertion (A) The function defined by $f(x)=\cos(x^2)$ is a continuous function.

Reason (R) The sine function is continuous in its domain i.e. $x\in R$.

$f(x)=[x-1]+|x-2|$, where $[\cdot]$ denotes the greatest integer function.

Assertion (A) $f(x)$ is discontinuous at $x=2$.

Reason (R) $f(x)$ is non derivable at $x=2$.

Assertion (A) $f(x)=|x-3|$ is continuous at $x=0$.

Reason (R) $f(x)=|x-3|$ is differentiable at $x=0$.

Assertion (A) Every differentiable function is continuous but converse is not true.

Reason (R) Function $f(x)=|x|$ is continuous.

Assertion (A) If $f(x)=\frac{\sin(ax+b)}{\cos(cx+d)}$, then $f'(x)=a\cos(ax+b)\sec(cx+d)+c\sin(ax+b)\tan(cx+d)\sec(cx+d)$

Reason (R) If $f(x)=\frac{u}{v}$, then $f'(x)=\frac{vu'-uv'}{v^2}$.

Assertion (A) $\frac{d}{dx}e^{\sin x}=e^{\sin x}(\cos x)$

Reason (R) $\frac{d}{dx}e^x=e^x$

Assertion (A) $\frac{d}{dx}\left(\sqrt{e^{\sqrt{x}}}\right)=\frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$.

Reason (R) $\frac{d}{dx}[\log(\log(x))]=\frac{1}{x\log x}$, $x>1$

Assertion (A) If $f(x)=\log x$, then $f''(x)=-\frac{1}{x^2}$.

Reason (R) If $y=x^3\log x$, then $\frac{d^2y}{dx^2}=x(5+6\log x)$.

10. (d) 11. (d) 12. (c)

Assertion Given, $f(x)=\sqrt[3]{x}$ or $f(x)=(x)^{1/3}$

Now, we check the continuity of the function at $x=0$.

$$\text{LHL}=f(0-0)=\lim_{h\to 0} f(0-h)$$
$$=\lim_{h\to 0}(0-h)^{1/3}$$
$$=(0-0)^{1/3}=0$$

$$\text{RHL}=f(0+0)=\lim_{h\to 0} f(0+h)$$
$$=\lim_{h\to 0}(0+h)^{1/3}=(0+0)^{1/3}=0$$

and $f(0)=(0)^{1/3}=0$

$\because$ LHL = RHL = $f(0)$

So, function is continuous at $x=0$.

Reason Given, $f(x)=[x]$, which is greatest integer function.

We know that, the greatest integer function is continuous for all x except integer values of x.

So, $f(x)=[x]$ is continuous at $x=2.99$.

Assertion Here, $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 \sin \frac{1}{x}.$$

Putting $x = 0 - h$ as $x \to 0^-$, $h \to 0$

$$\therefore \lim_{h \to 0} (0-h)^2 \sin\left(\frac{1}{0-h}\right) = \lim_{h \to 0}\left(-h^2 \sin \frac{1}{h}\right) \quad [\because \sin(-\theta) = -\sin\theta]$$

$$= -0 \times \sin(\infty)$$
$$= -0 \times (\text{a finite value between } -1 \text{ and } 1)$$
$$= 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 \sin \frac{1}{x}$$

Putting $x = 0 + h$, as $x \to 0^+$, $h \to 0$

$$\therefore \lim_{h \to 0} (0+h)^2 \sin\left(\frac{1}{0+h}\right) = \lim_{h \to 0} h^2 \sin \frac{1}{h}$$

$$= 0 \times \sin(\infty)$$
$$= 0 \times (\text{a finite value between } -1 \text{ and } 1)$$
$$= 0$$

Also, $f(0) = 0$

$\therefore \quad \text{LHL} = \text{RHL} = f(0)$.

Thus, $f(x)$ is continuous at $x = 0$.

Reason Here, $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (\sin x - \cos x)$$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$$\therefore \lim_{h \to 0} [\sin(0-h) - \cos(0-h)]$$
$$= \lim_{h \to 0} (-\sin h - \cos h)$$
$$= 0 - 1 = -1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (\sin x - \cos x)$$

Putting $x = 0 + h$ as $x \to 0^+$ when $h \to 0$

$$\therefore \lim_{h \to 0} [\sin(0+h) - \cos(0+h)]$$
$$= \lim_{h \to 0} (\sin h - \cos h)$$
$$= 0 - 1 = -1$$

Also, $f(0) = -1$

$\therefore \quad \text{LHL} = \text{RHL} = f(0)$.

Thus, $f(x)$ is continuous at $x = 0$.

We know, when $x < 0$, $f(x) = \sin x - \cos x$ is continuous and when $x > 0$, $f(x) = \sin x - \cos x$ is also continuous.

Hence, $f(x)$ is continuous for all values of x.

Assertion We know that,

If $f(a) = \lim_{x \to a} f(x)$, then $f(x)$ is continuous at $x = a$, while both hand must exist.

Reason If $f(x)$ is continuous at a point, then it is not necessary that $\frac{1}{f(x)}$ is also continuous at that point.

e.g. $f(x) = x$ is continuous at $x = 0$ but $f(x) = \frac{1}{x}$ is not continuous at $x = 0$.

Assertion $f(x) = \begin{cases} \sin \pi x, & x < 1 \\ 0, & x = 1 \\ -\frac{\sin(x-1)}{x}, & x > 1 \end{cases}$

$$\text{Also, LHL} = \lim_{x \to 1^-} f(x)$$
$$= \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \sin(\pi - \pi h)$$
$$= \lim_{h \to 0} \sin(\pi h) = \sin 0 = 0$$

$$\text{RHL} = \lim_{x \to 1^+} f(x)$$
$$= \lim_{h \to 0} f(1+h)$$
$$= \lim_{h \to 0} \frac{-\sin(1+h-1)}{(1+h)}$$
$$= -\lim_{h \to 0} \frac{\sin h}{1+h} = 0$$

and $\quad f(1) = 0$

$\therefore \quad \text{LHL} = \text{RHL} = f(1)$

$\Rightarrow f(x)$ is continuous at $x = 1$.

$\therefore$ Assertion is false.

Reason It is clear that $f(1) = 0$

$\therefore$ Reason is true.

Assertion Here, $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \le x \le 1 \\ 4x, & \text{if } x > 1 \end{cases}$

For $x < 0$, $f(x) = 2x$; $0 < x < 1$, $f(x) = 0$ and $x > 1$, $f(x) = 4x$ are polynomial and constant functions, so it is continuous in the given interval.

So, we have to check the continuity at $x = 0$ and 1.

At $x = 0$,

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (2x)$$

Putting $x = 0 - h$ as $x \to 0^-, h \to 0$

$\therefore \lim_{h\to 0} [2(0-h)] = \lim_{h\to 0} (-2h) = -2 \times 0 = 0,$

$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (0) = 0$

Also, $f(0) = 0$

$\therefore \quad \text{LHL} = \text{RHL} = f(0)$

Thus, $f(x)$ is continuous at $x = 0$.

At $x = 1$,

$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (0) = 0,$

$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (4x)$

Putting $x = 1 + h$ as $x \to 1^+$ when $h \to 0$

$\text{RHL} = \lim_{h\to 0} 4(1+h) = \lim_{h\to 0} (4+4h)$

$= 4 + 4 \times 0 = 4$

$\therefore$ LHL $\neq$ RHL.

Thus, $f(x)$ is continuous everywhere except at $x = 1$.

Assertion $\text{LHL} = \lim_{x\to 0^-} f(x)$

$= \lim_{x\to 0} (x + \pi) = \pi$

$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} \pi \cos x$

$= \pi \cos(0) = \pi$

Also, $f(0) = \pi \cos(0) = \pi$

Hence, $f(x)$ is continuous at $x = 0$.

$\therefore$ Assertion is true.

Reason Now, for $x = \frac{\pi}{2}$

$\text{LHL} = \lim_{x\to \pi/2^-} f(x) = \lim_{x\to \pi/2} \pi \cos x$

$= \pi \cos \frac{\pi}{2} = 0$

$\text{RHL} = \lim_{x\to \pi/2^+} f(x) = \lim_{x\to \pi/2} \left(x - \frac{\pi}{2}\right)^2$

$= \left(\frac{\pi}{2} - \frac{\pi}{2}\right)^2 = 0$

Also, $f\left(\frac{\pi}{2}\right) = \pi \cos \frac{\pi}{2} = 0$

Hence, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$\therefore$ Reason is true.

Assertion We have, $f(x) = |\cos x|$

$$= \begin{cases} \cos x, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Continuity at $x = 0$,

$\text{LHL} = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \cos(0-h) = \cos 0 = 1$

$\text{RHL} = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \cos(0+h)$

$= \lim_{h\to 0} \cos h = \cos 0 = 1$

and $f(0) = 1$

$\therefore$ LHL = RHL = $f(0)$

So, $f(x)$ is continuous at $x = 0$.

Hence, $f(x)$ is continuous everywhere.

Reason We have, $f(x) = \cos|x|$

$$= \begin{cases} \cos x, & x \geq 0 \\ \cos(-x), & x < 0 \end{cases}$$

$$= \begin{cases} \cos x, & x \geq 0 \\ \cos x, & x < 0 \end{cases}$$

$= \cos x, \; x \in R$

But $\cos x$ is always continuous in their domain.

Hence, $f(x)$ is continuous everywhere.

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Assertion We have, $f(x) = \cos(x^2)$

At $x = c$,

$\text{LHL} = \lim_{h\to 0} \cos(c-h)^2 = \cos c^2$

$\text{RHL} = \lim_{h\to 0} \cos(c+h)^2 = \cos c^2$

and $f(c) = \cos c^2$

$\therefore$ LHL = RHL = $f(c)$

So, $f(x)$ is continuous at $x = c$.

Hence, $f(x)$ is continuous for every value of x.

Hence, both Assertion and Reason are true and Reason is not the correct explanation of Assertion.

Assertion

$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$

$= \lim_{h\to 0} [2-h-1] + |2-h-2|$

$= \lim_{h\to 0} [1-h] + |-h| = \lim_{h\to 0} (0+h) = 0$

and $\quad f(2) = [2-1] + |2-2| = [1] + 0 = 1$

$\therefore \quad \text{LHL} \neq f(2)$

$\Rightarrow f(x)$ is discontinuous at $x = 2$.

Reason

$$Lf'(2) = \lim_{h\to0} \frac{f(2-h)-f(2)}{-h}$$

$$= \lim_{h\to0} \frac{[2-h-1]+|2-h-2|-[2-1]-|2-2|}{-h}$$

$$= \lim_{h\to0} \frac{0+h-1-0}{-h} \qquad [\because \lim_{h\to0}[1-h]=0]$$

$$= \lim_{h\to0}\left(1-\frac{1}{h}\right) = -\infty \qquad \text{(not defined)}$$

$\therefore$ $f(x)$ is not differentiable at $x=2$.

Hence, both Assertion and Reason are true and Reason is not a correct explanation of Assertion.

Assertion $\because$ $f(x) = |x-3| = \begin{cases} x-3, & x \geq 3 \\ 3-x, & x<3 \end{cases}$

$$\therefore \quad \text{LHL} = \lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h)$$

$$= \lim_{h\to0}(3+h) = 3$$

$$\text{RHL} = \lim_{x\to0^+} f(x) = \lim_{h\to0} f(0+h)$$

$$= \lim_{h\to0}(3-h) = 3$$

and $f(0) = 3-0 = 3$

$\Rightarrow$ LHL = RHL = $f(0)$

So, $f(x)$ is continuous at $x=0$.

Reason Now, LHD = $f'(0^-)$

$$= \lim_{h\to0} \frac{f(0)-f(0-h)}{h}$$

$$= \lim_{h\to0} \frac{3-(3-h)}{h} = 1$$

and RHD = $f'(0^+) = \lim_{h\to0} \dfrac{f(0+h)-f(0)}{h}$

$$= \lim_{h\to0} \frac{3+h-3}{h} = 1$$

$\Rightarrow$ LHD = RHD

$\therefore$ $f(x)$ is differentiable at $x=0$.

Hence, both Assertion and Reason are true.

Assertion It is a true statement.

Reason We have, $f(x) = |x|$

At $x=0$,

$$\text{LHL} = \lim_{h\to0^-} \frac{f(0-h)-f(0)}{-h}$$

$$= \lim_{h\to0^-} \frac{|0-h|-0}{-h}$$

$$= \lim_{h\to0^-} \frac{h}{-h} = -1$$

and $\text{RHL} = \lim_{h\to0^+} \dfrac{f(0+h)-f(0)}{h}$

$$= \lim_{h\to0^+} \frac{|0+h|-0}{h} = \lim_{h\to0^+} \frac{h}{h} = 1$$

Here, LHD $\neq$ RHD, hence $f(x)$ is not continuous at $x=0$.

Let $y = \dfrac{\sin(ax+b)}{\cos(cx+d)}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin(ax+b)}{\cos(cx+d)}\right)$$

$$= \frac{\cos(cx+d)\dfrac{d}{dx}\{\sin(ax+b)\} - \sin(ax+b)\dfrac{d}{dx}\cos(cx+d)}{[\cos(cx+d)]^2}$$

[by quotient rule]

$$= \frac{\cos(cx+d)\cos(ax+b)(a+0) + \sin(ax+b)\sin(cx+d)(c+0)}{\cos^2(cx+d)}$$

[by chain rule,

$\dfrac{d}{dx}\sin(ax+b) = \cos(ax+b)\dfrac{d}{dx}(ax+b)$

$= \cos(ax+b)\times(a\times1+0)$

$\dfrac{d}{dx}\cos(cx+d) = -\sin(cx+d)\dfrac{d}{dx}(cx+d)$

$= -\sin(cx+d)\times(c\times1+0)$]

$$= \frac{a\cos(cx+d)\cos(ax+b) + c\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$$

$$= \frac{a\cos(cx+d)\cos(ax+b)}{\cos^2(cx+d)} + \frac{c\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$$

$$= \frac{a\cos(ax+b)}{\cos(cx+d)} + \frac{c\sin(ax+b)\sin(cx+d)}{\cos(cx+d)\cos(cx+d)}$$

$$= a\cos(ax+b)\sec(cx+d) + c\sin(ax+b)\tan(cx+d)\sec(cx+d)$$

Assertion Let $y = e^{\sin x}$.

Using chain rule, we have

$$\frac{dy}{dx} = e^{\sin x}\cdot(\cos x)$$

$$= \cos x\, e^{\sin x}$$

Reason $\frac{d}{dx}(e^x) = e^x \cdot \frac{d(x)}{dx} = e^x \times 1 = e^x$

Hence, both Assertion and Reason are true, but Reason is the correct explanation of Assertion.

Assertion Let $y = (e^{\sqrt{x}})^{1/2}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2}(e^{\sqrt{x}})^{\frac{1}{2}-1} \frac{d}{dx} e^{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(e^{\sqrt{x}})^{-\frac{1}{2}} \cdot e^{\sqrt{x}} \cdot \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{e^{\sqrt{x}}}} \times \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{4\sqrt{x}\sqrt{e^{\sqrt{x}}}} = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$$

Reason Let $y = \log(\log x)$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\log(\log x)) = \frac{1}{\log x}\left\{\frac{d}{dx}(\log x)\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}, \ x > 1$$

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Assertion Let $y = \log x$

On differentiating twice w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{x}$$

and $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

Reason Let $y = x^3 \log x$

On differentiating twice w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 \log x)$$

$$= x^3 \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^3)$$

$$= x^3\left(\frac{1}{x}\right) + (\log x)(3x^2)$$

$$= x^2(1 + 3\log x)$$

[using product rule]

and $$\frac{d^2y}{dx^2} = \frac{d}{dx}\{x^2(1 + 3\log x)\}$$

$$= x^2\left(0 + \frac{3}{x}\right) + (1 + 3\log x)(2x)$$

$$= 3x + 2x(1 + 3\log x)$$

$$= x(5 + 6\log x)$$

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

also continuous in R.
Hence A is true.
R is the correct explanation of A.

Assertion (A): $f(x) = \tan^2 x$ is continuous at $x = \frac{\pi}{2}$.

Reason (R): $g(x) = x^2$ is continuous at $x = \frac{\pi}{2}$.

Ans. Option (D) is correct.

Explanation: $g(x) = x^2$ is a polynomial function. It is continuous for all $x \in R$.
Hence R is true.

$f(x) = \tan^2 x$ is not defined when $x = \frac{\pi}{2}$.

Therefore $f\left(\frac{\pi}{2}\right)$ does not exist and hence $f(x)$ is not continuous at $x = \frac{\pi}{2}$.

A is false.

Consider the function $f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$

which is continuous at $x = 0$.

Assertion (A): The value of k is -3.

Hence R is true.
Since f is continuous at $x = 0$,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

Here $f(0) = 3$,

$$\text{LHL} = \lim_{x\to 0^-} f(x)$$

$$= \lim_{x\to 0^-} \frac{kx}{|x|} = \lim_{x\to 0^-} \frac{kx}{-x} = -k$$

$\therefore$ $-k = 3$ or $k = -3$.

Hence A is true.
R is the correct explanation of A.

Consider the function

$$f(x) = \begin{cases} \frac{x^2+3x-10}{x-2}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$$

which is continuous at $x = 2$.

Assertion (A): The value of k is 0.

Reason (R): $f(x)$ is continuous at $x = a$, if

$$\lim_{x\to a} f(x) = f(a).$$

Ans. Option (D) is correct.

Explanation:
$f(x)$ is continuous at $x = a$, if $\lim_{x\to a} f(x) = f(a)$.
$\therefore$ R is true.

$$\lim_{x\to 2} f(x) = f(2) = k$$

$$\lim_{x\to 2} \frac{(x+5)(x-2)}{x-2} = k$$

$\therefore$ $k = 7$

Hence A is false.

Assertion (A): $|\sin x|$ is continuous at $x = 0$.

Reason (R): $|\sin x|$ is differentiable at $x = 0$.

Ans. Option (C) is correct.

Explanation: Since sin x and $|x|$ are continuous functions in R, $|\sin x|$ is continuous at $x = 0$.

Hence A is true.

$$|\sin x| = \begin{cases} -\sin x, & \text{if } x < 0 \\ \sin x, & \text{if } x \geq 0 \end{cases}$$

$$f(0) = |\sin 0| = 0$$

$$\text{LHD} = f'(0^-) = \lim_{x\to 0} \frac{-\sin x - 0}{x} = -1$$

$$\text{RHD} = f'(0^+) = \lim_{x\to 0} \frac{\sin x - 0}{x} = 1$$

At $x = 0$, LHD $\neq$ RHD.

So $f(x)$ is not differentiable at $x = 0$.

Hence R is false.

Assertion (A): $f(x) = [x]$ is not differentiable at $x = 2$.

Reason (R): $f(x) = [x]$ is not continuous at $x = 2$.

Ans. Option (A) is correct.

Explanation: $f(x) = [x]$ is not continuous when x is an integer.

So $f(x)$ is not continuous at $x = 2$. Hence R is true.

A differentiable function is always continuous.

Since $f(x) = [x]$ is not continuous at $x = 2$, it is also not differentiable at $x = 2$.

Hence A is true.

R is the correct explanation of A.

Assertion (A): A continuous function is always differentiable.

Reason (R): A differentiable function is always continuous.

Ans. Option (D) is correct.

Explanation: The function $f(x)$ is differentiable at $x = a$, if it is continuous at $x = a$ and

LHD = RHD at $x = a$.

A differentiable function is always continuous. Hence R is true.

A continuous function need not be always differentiable.

For example, $|x|$ is continuous at $x = 0$, but not differentiable at $x = 0$.

Hence A is false.

Assertion (A): If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, then $\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$

Reason (R): $\sin^{-1}(6x\sqrt{1-9x^2}) = 3\sin^{-1}(2x)$

Ans. Option (C) is correct.

Explanation:

put $3x = \sin\theta$ or $\theta = \sin^{-1}3x$

$$y = \sin^{-1}(6x\sqrt{1-9x^2}) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}3x$$

$$\therefore \quad \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

A is true. R is false.